**Title: Newton's Law of Cooling: Continuous Model and Euler's Method**

**Brief Overview:**

Students will explore Newton's Law of Cooling experimentally using the CBL. Additionally, they will explore the mathematical model for Newton's Law of Cooling using Euler's Method.

**Links to NCTM Standards:**

- **Mathematics as Problem Solving**
  Students will investigate the cooling of water and demonstrate the ability to collect and make inferences from their data. They will also demonstrate their ability to solve mathematical problems through the use of graphical analysis and data analysis.

- **Mathematics as Communication**
  Students will explain procedures and processes orally. They will express mathematical assumptions both orally and in writing.

- **Mathematics as Reasoning**
  Students will make conjectures concerning data, their graphs and changes in data.

- **Mathematical Connections**
  Students will make connections between mathematical functions and experimental data.

- **Algebra**
  Students will solve differential equations and represent situations with variable quantities.

- **Calculus**
  Students will apply the slope of a graph to data analysis. They will analyze graphs of transcendental functions.

**Grade/Level:**

Grades 11-12 (Calculus AB and BC)

**Duration/Length:**

This activity will take 2-3 days.

**Prerequisite Knowledge:**

Students should have working knowledge of the following skills:

- The exponential function
- Spreadsheets
- Graphing Calculators and the CBL
- Slope of functions
- Solving differential equations

**Objectives:**

Students will:

- work cooperatively in groups.
- collect and analyze data from the CBL.
- solve a differential equation.
- create a spreadsheet to model the cooling behavior of water.
- compare the experimental data to the mathematical model.

**Materials/Resources/Printed Materials:**

- Heat source (microwave, hot plate)
- Container for water
- CBL and TI Calculator
- Computers with spreadsheet software
- Teacher hand-outs
- Student hand-outs

**Development/Procedures:**

- Students will set up the CBL and collect temperature data as boiling water cools.
- Students will identify the nature of the graph of their data.
- Students will solve the differential equations that model the physical properties of cooling liquids.
- Students will use a spreadsheet to generate values that will be analyzed using Euler's Method.

**Evaluation:**

- Students will successfully complete the enclosed worksheets. Solutions are provided for the instructor.

**Extension/Follow Up:**

- Students may use data collected in the CBL lab to generate the exponential function for Newton's Law of Cooling and to verify the accuracy of their mathematical model.
- Students may analyze the cooling behavior of substances other than water, including solids.

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**Newton's Law of Cooling: Data Collection Using the CBL** **Name ____________**

## I. Set up and data collection

Heat a container of water to approximately boiling temperature, using a hot pot or microwave if available. Connect the TI calculator to the CBL unit, turning on both. Choose the program CHILL (from Real-World Math with the CBL™ System by C. Brueningsen et al, 1994) from the calculator's program file. Run the program and follow its directions as given on your calculator screen. When you are finished, examine the graph on your calculator's window.

## II. Analysis

### A. Graph Analysis

1. What type of function does this graph appear to be?

2. What is the ambient room temperature? _________ What temperature does the graph approach as time increases? ____________

### B. Analysis of the Mathematical Model using Calculus

Newton's Law of Cooling states that the change in temperature of a substance is proportional to the difference between its temperature and the ambient room temperature.

The differential equation representing this model is:

$$\frac{dy}{dt} = k\,(y-T)$$

where y represents temperature of the liquid, t represents time and T is a constant, representing ambient temperature.

1. Solve the differential equation above, using your data in order to calculate the constants. Use your initial point when t = 0 for your first ordered pair (t,y). Choose one other ordered pair from your data. Show all work.

2. Enter the equation generated in #1 above into the graphing calculator. How does your model compare with the actual data? How do you account for any differences?

**<u>Newton's Law of Cooling</u>** **Name ___________________**

**Spreadsheet, Euler's Method Exercise**

Newton's Law of Cooling states that the change in temperature of a substance is proportional to the difference between its temperature and the ambient room temperature.

The differential equation representing this model is:

$$\frac{dy}{dt} = k\,(y-T)$$

where y represents temperature of the liquid, t represents time and T is a constant, representing ambient temperature

According to Euler, $\frac{dy}{dt}$ can be approximated, *for small values of h*, by the slope of the line between two consecutive points as follows:

$$\frac{y(t+h) - y(t)}{h} = k\,(y(t)-T)$$ where y(t) represents the temperature at time t.

Solving for y(t+h):

$$y(t + h) = hk\,(y(t) - T) + y(t)$$

Expressing the above equation recursively:

$$\mathbf{Y_{i+1} = hk\,(Y_i - T) + Y_i}$$

## I. Directions for Spreadsheet Activity:

| | | | | |
|---|---|---|---|---|
| **Newton's Law of Cooling** | | | | |
| | | | | |
| T = ambient temperature (degrees F) | 60 | | | |
| h = change in time (minutes) | 1 | | | |
| k = rate of cooling (rate of change/unit of time) | -0.02877 | | | |
| Y= temperature at time t (degrees F) | Yo = | 100 | | |
| | | | | |
| **Time (Minutes)** | **Temperature (degrees F)** | | | |
| 0 | 100 | | | |
| 1 | (Your computed value for $Y_{i+1}$) | | | |

1. Using the example above, enter the documentation for the spreadsheet, including:
   a. Title
   b. Names of variables and constants
   c. Values of variables and constants
   d. Units of measure
   e. Column labels

2. Fill the Time column with 100 cells (or more, if necessary according to your value of h).

3. Enter the recursive formula for Temperature, $Y_{i+1}$ and fill down your Temperature column.

4. Graph temperature vs. time (if your software supports this option.) For example, Microsoft Excel allows you to insert a chart with line graphs.

## II. Analysis of Results:

1. What is the significance of the asymptote of your graph? Explain your answer.

2. How would your results differ if $Y_o$ were 60 degrees? Explain your answer.

3. Change your parameters from the given sample to the following, and answer the questions, which follow.
   a. $T = 90^o$ , $Y_o = 1500^o$ , $k = -.31403$, $h = 1$ minute.

      i. What does the asymptote of your graph represent? Explain your answer.

      ii. Find Y at time = 1 minute

      iii. Reduce the value of h. How does this new value affect your results? Complete the table below for various values of h.

| h (minutes) | Value of Y at t = 1 min. |
|---|---|
| | |
| | |
| | |

   b. $T = 90^o$, $Y_o = 45^o$, $k = -.03$, $h = 1$ minute.

      i. What does the asymptote of your graph represent? Explain your answer.

      ii. Explain the shape of this graph. Why is it different from previous graphs?

      iii. Find Y at time = 1 minute.

      iv. Reduce the value of h. How does this new value affect your results? Complete the table below for various values of h.

| h (minutes) | Value of Y at t = 1 min. |
|---|---|
| | |
| | |
| | |

**<u>Newton's Law of Cooling Extension</u>:** **Name __________________**

**Comparison of Collected Data to Spreadsheet Model**

Change the initial parameters on your spreadsheet. Use the *values generated* during the CBL lab for ambient temperature (T) and initial temperature ($Y_o$). Use your *computed values* for rate of cooling (k).

**Explorations:**

1. Enter the same value for h (change in time) as used by the CBL program. Compare the values on the calculator to the values on the spreadsheet.

    a. How do these values compare?

    b. How do you account for differences between the mathematical model and the data-generated model?

2. Change the values of h and watch the behavior of the temperatures in the spreadsheet.

    a. What happens to the values of the temperature as h becomes larger?

    b. What happens to the values of the temperature as h becomes smaller?

3. Input various values for the initial temperature ($Y_o$). How does the proximity of $Y_o$ to the ambient temperature affect the graph?

**<u>Newton's Law of Cooling:</u>** **Name: <u>SOLUTION KEY</u>**
**<u>Data Collection Using the CBL</u>**

**I. Set up and data collection**

**TEACHER'S NOTE: THERE ARE MANY OTHER SOURCES FOR CBL PROGRAMS FOR NEWTON'S LAW OF COOLING. SEE THE TI INTERNET SITE (www.ti.com) FOR A LIST OF RESOURCES.**

**II. Analysis**

**A. Graph Analysis**

1. What type of function does this graph appear to be? **EXPONENTIAL**

2. What is the ambient room temperature? **<u>(ANSWERS VARY)</u>** What temperature does the graph approach as time increases? **<u>THE AMBIENT TEMPERATURE</u>**

**B. Analysis of the Mathematical Model using Calculus**

**1. Solution worked for initial values of T = 60$^o$ , Y$_o$ = 100$^o$ , and Y(10) = 90$^o$. Students' results will vary according to their own data.**

$$\frac{dy}{dt} = k\,(y-T)$$ Now separate the variables:

$$\frac{dy}{y-T} = k\,dt$$ Now integrate both sides:

$$\int \frac{dy}{y-T} = \int k\,dt$$

$$\ln|\,y-T\,| = kt + C$$

$$e^{kt+C} = y - T$$

$$C\,e^{kt} = y - T$$

$$Y = Ce^{kt} + T$$

Now solve for C and k, using 2 ordered pairs (t,y) from the data gathered.

From values given above, $k = .1 * \ln(.75) \approx$ **-.02877** and **C = 40**

2. Enter the equation generated in #1 above into the graphing calculator. How does your model compare with the actual data? How do you account for any differences?

**THE GRAPHS SHOULD BE VERY SIMILAR. DIFFERENCES MAY BE DUE TO THE FACT THAT THE SOLUTION OF THE DIFFERENTIAL EQUATION IS A CONTINUOUS FUNCTION, WHEREAS THE CBL EXPERIMENT MEASURES AT DISCRETE TIME INTERVALS.**

**<u>Newton's Law of Cooling:</u>** **Name: <u>SOLUTION KEY</u>**
**<u>Spreadsheet, Euler's Method Exercise</u>**

**I. Directions for Spreadsheet Activity: SEE ATTACHED SPREADSHEET**

**II. Analysis of Results:**

1. What is the significance of the asymptote of your graph? Explain your answer.

**THE AMBIENT TEMPERATURE. THE SUBSTANCE WILL EVENTUALLY REACH ROOM TEMPERATURE (UNLIKE THE MATHEMATICAL MODEL, WHOSE FUNCTION IS ASYMPTOTIC.)**

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2. How would your results differ if $Y_o$ were 60 degrees? Explain your answer.

**THE GRAPH WOULD BE A CONSTANT FUNCTION AT THE AMBIENT TEMPERATURE. THERE IS NO DIFFERENCE BETWEEN THE INITIAL TEMPERATURE AND AMBIENT TEMPERATURE.**

3. Change your parameters from the given sample to the following, and answer the questions which follow.
   a. $T = 90^o$ , $Y_o = 1500^o$ , $k = -.31403$, $h = 1$ minute.

      i. What does the asymptote of your graph represent? Explain your answer.

**THE AMBIENT TEMPERATURE. THE SUBSTANCE WILL EVENTUALLY REACH ROOM TEMPERATURE (UNLIKE THE MATHEMATICAL MODEL, WHOSE FUNCTION IS ASYMPTOTIC.)**

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      ii. Find Y at time = 1 minute.

      iii. Change the value of h. How does this new value affect your results? Complete the table below for various values of h.

| h (minutes) | Value of Y at t = 1 min. |
|---|---|
| 1 | 1057.218 |
| .1 | 1114.826 |
| .01 | 1119.492 |

b. $T = 90^o$, $Y_o = 45^o$, $k = -.03$, $h = 1$ minute.

i. What does the asymptote of your graph represent? Explain your answer.

**THE AMBIENT TEMPERATURE. THE SUBSTANCE WILL EVENTUALLY REACH ROOM TEMPERATURE (UNLIKE THE MATHEMATICAL MODEL, WHOSE FUNCTION IS ASYMPTOTIC.)**

ii. Explain the shape of this graph. Why is it different from previous graphs?

**BECAUSE THE INITIAL TEMPERATURE IS BELOW AMBIENT TEMPERATURE, THE SUBSTANCE WARMS RATHER THAN COOLS. THEREFORE THE FUNCTION IS INCREASING.**

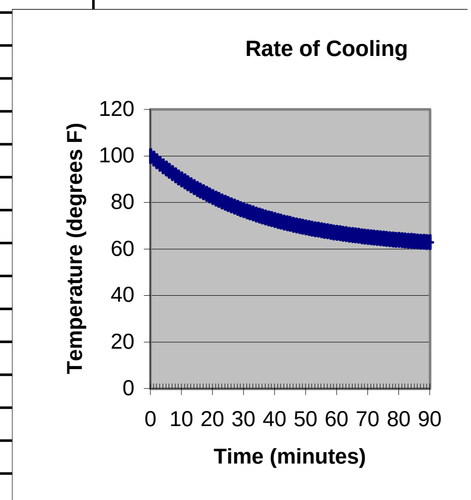
iii. Find Y at time = 10 minute.

iv. Increase the value of h. How does this new value affect your results? Complete the table below for various values of h.

| h (minutes) | Value of Y at t = 10 min |
|---|---|
| 1 | 56.816 |
| 2 | 56.974 |
| 5 | 57.488 |

Newton's Law of Cooling

| | | |
|---|---|---|
| | | |
| T = ambient temperature (degrees F) | 60 | |
| h = change in time (minutes) | 1 | |
| k = rate of change/unit of time | -0.02877 | |
| Y=temperature at time t (degrees F) | $Y_o$= | 100 |
| | | |
| Time (minutes) | Temperature (degrees F) | |
| 0 | 100 | |
| 1 | 98.8492 | |
| 2 | 97.73150852 | |
| 3 | 96.64597302 | |
| 4 | 95.59166837 | |
| 5 | 94.56769607 | |
| 6 | 93.57318346 | |
| 7 | 92.60728297 | |
| 8 | 91.66917144 | |
| 9 | 90.75804938 | |
| 10 | 89.8731403 | |
| 11 | 89.01369005 | |
| 12 | 88.17896619 | |
| 13 | 87.36825733 | |
| 14 | 86.58087257 | |
| 15 | 85.81614086 | |
| 16 | 85.07341049 | |
| 17 | 84.35204847 | |
| 18 | 83.65144004 | |
| 19 | 82.97098811 | |
| 20 | 82.31011278 | |
| 21 | 81.66825083 | |
| 22 | 81.04485526 | |
| 23 | 80.43939477 | |
| 24 | 79.85135338 | |
| 25 | 79.28022995 | |
| 26 | 78.72553773 | |
| 27 | 78.18680401 | |
| 28 | 77.66356966 | |
| 29 | 77.15538876 | |
| 30 | 76.66182823 | |
| 31 | 76.18246743 | |
| 32 | 75.71689784 | |
| 33 | 75.26472269 | |
| 34 | 74.82555662 | |
| 35 | 74.39902535 | |
| 36 | 73.98476539 | |
| 37 | 73.58242369 | |
| 38 | 73.19165736 | |
| 39 | 72.81213338 | |
| 40 | 72.4435283 | |
| 41 | 72.08552799 | |
| 42 | 71.73782735 | |
| 43 | 71.40013006 | |



| | |
|---|---|
| 44 | 71.07214832 |
| 45 | 70.75360261 |
| 46 | 70.44422146 |
| 47 | 70.14374121 |
| 48 | 69.85190578 |
| 49 | 69.56846645 |
| 50 | 69.29318167 |
| 51 | 69.02581683 |
| 52 | 68.76614408 |
| 53 | 68.51394212 |
| 54 | 68.268996 |
| 55 | 68.03109699 |
| 56 | 67.80004233 |
| 57 | 67.57563511 |
| 58 | 67.35768409 |
| 59 | 67.14600352 |
| 60 | 66.940413 |
| 61 | 66.74073731 |
| 62 | 66.5468063 |
| 63 | 66.35845468 |
| 64 | 66.17552194 |
| 65 | 65.99785218 |
| 66 | 65.82529397 |
| 67 | 65.65770026 |
| 68 | 65.49492822 |
| 69 | 65.33683914 |
| 70 | 65.18329828 |
| 71 | 65.03417479 |
| 72 | 64.88934158 |
| 73 | 64.74867522 |
| 74 | 64.61205583 |
| 75 | 64.47936699 |
| 76 | 64.3504956 |
| 77 | 64.22533184 |
| 78 | 64.10376904 |
| 79 | 63.98570361 |
| 80 | 63.87103492 |
| 81 | 63.75966524 |
| 82 | 63.65149967 |
| 83 | 63.54644603 |
| 84 | 63.44441477 |
| 85 | 63.34531896 |
| 86 | 63.24907414 |
| 87 | 63.15559827 |
| 88 | 63.06481171 |
| 89 | 62.97663708 |
| 90 | 62.89099923 |
| 91 | 62.80782518 |
| 92 | 62.72704405 |
| 93 | 62.64858699 |
| 94 | 62.57238715 |
| 95 | 62.49837957 |

| | |
|---|---|
| 96 | 62.42650119 |
| 97 | 62.35669075 |
| 98 | 62.28888875 |
| 99 | 62.22303743 |
| 100 | 62.15908064 |

Newton's Law of Cooling **SOLUTION KEY**

| | | |
|---|---|---|
| | | |
| T = ambient temperature (degrees F) | 60 | |
| h = change in time (minutes) | 1 | |
| k = rate of change/unit of time | -0.02877 | |
| Y=temperature at time t (degrees F) | $Y_o$= | 100 |
| | | |
| Time (minutes) | Temperature (degrees F) | |
| 0 | =$C$6 | |
| =A9+$B$4 | =$B$4*$B$5*(B9-$B$3)+B9 | |
| =A10+$B$4 | =$B$4*$B$5*(B10-$B$3)+B10 | |
| =A11+$B$4 | =$B$4*$B$5*(B11-$B$3)+B11 | |
| =A12+$B$4 | =$B$4*$B$5*(B12-$B$3)+B12 | |
| =A13+$B$4 | =$B$4*$B$5*(B13-$B$3)+B13 | |
| =A14+$B$4 | =$B$4*$B$5*(B14-$B$3)+B14 | |
| =A15+$B$4 | =$B$4*$B$5*(B15-$B$3)+B15 | |
| =A16+$B$4 | =$B$4*$B$5*(B16-$B$3)+B16 | |
| =A17+$B$4 | =$B$4*$B$5*(B17-$B$3)+B17 | |
| =A18+$B$4 | =$B$4*$B$5*(B18-$B$3)+B18 | |
| =A19+$B$4 | =$B$4*$B$5*(B19-$B$3)+B19 | |
| =A20+$B$4 | =$B$4*$B$5*(B20-$B$3)+B20 | |
| =A21+$B$4 | =$B$4*$B$5*(B21-$B$3)+B21 | |
| =A22+$B$4 | =$B$4*$B$5*(B22-$B$3)+B22 | |
| =A23+$B$4 | =$B$4*$B$5*(B23-$B$3)+B23 | |
| =A24+$B$4 | =$B$4*$B$5*(B24-$B$3)+B24 | |
| =A25+$B$4 | =$B$4*$B$5*(B25-$B$3)+B25 | |
| =A26+$B$4 | =$B$4*$B$5*(B26-$B$3)+B26 | |
| =A27+$B$4 | =$B$4*$B$5*(B27-$B$3)+B27 | |
| =A28+$B$4 | =$B$4*$B$5*(B28-$B$3)+B28 | |
| =A29+$B$4 | =$B$4*$B$5*(B29-$B$3)+B29 | |
| =A30+$B$4 | =$B$4*$B$5*(B30-$B$3)+B30 | |
| =A31+$B$4 | =$B$4*$B$5*(B31-$B$3)+B31 | |
| =A32+$B$4 | =$B$4*$B$5*(B32-$B$3)+B32 | |
| =A33+$B$4 | =$B$4*$B$5*(B33-$B$3)+B33 | |
| =A34+$B$4 | =$B$4*$B$5*(B34-$B$3)+B34 | |
| =A35+$B$4 | =$B$4*$B$5*(B35-$B$3)+B35 | |
| =A36+$B$4 | =$B$4*$B$5*(B36-$B$3)+B36 | |
| =A37+$B$4 | =$B$4*$B$5*(B37-$B$3)+B37 | |
| =A38+$B$4 | =$B$4*$B$5*(B38-$B$3)+B38 | |
| =A39+$B$4 | =$B$4*$B$5*(B39-$B$3)+B39 | |
| =A40+$B$4 | =$B$4*$B$5*(B40-$B$3)+B40 | |
| =A41+$B$4 | =$B$4*$B$5*(B41-$B$3)+B41 | |
| =A42+$B$4 | =$B$4*$B$5*(B42-$B$3)+B42 | |
| =A43+$B$4 | =$B$4*$B$5*(B43-$B$3)+B43 | |
| =A44+$B$4 | =$B$4*$B$5*(B44-$B$3)+B44 | |
| =A45+$B$4 | =$B$4*$B$5*(B45-$B$3)+B45 | |
| =A46+$B$4 | =$B$4*$B$5*(B46-$B$3)+B46 | |
| =A47+$B$4 | =$B$4*$B$5*(B47-$B$3)+B47 | |
| =A48+$B$4 | =$B$4*$B$5*(B48-$B$3)+B48 | |
| =A49+$B$4 | =$B$4*$B$5*(B49-$B$3)+B49 | |
| =A50+$B$4 | =$B$4*$B$5*(B50-$B$3)+B50 | |
| =A51+$B$4 | =$B$4*$B$5*(B51-$B$3)+B51 | |

| | |
|---|---|
| =A52+$B$4 | =$B$4*$B$5*(B52-$B$3)+B52 |
| =A53+$B$4 | =$B$4*$B$5*(B53-$B$3)+B53 |
| =A54+$B$4 | =$B$4*$B$5*(B54-$B$3)+B54 |
| =A55+$B$4 | =$B$4*$B$5*(B55-$B$3)+B55 |
| =A56+$B$4 | =$B$4*$B$5*(B56-$B$3)+B56 |
| =A57+$B$4 | =$B$4*$B$5*(B57-$B$3)+B57 |
| =A58+$B$4 | =$B$4*$B$5*(B58-$B$3)+B58 |
| =A59+$B$4 | =$B$4*$B$5*(B59-$B$3)+B59 |
| =A60+$B$4 | =$B$4*$B$5*(B60-$B$3)+B60 |
| =A61+$B$4 | =$B$4*$B$5*(B61-$B$3)+B61 |
| =A62+$B$4 | =$B$4*$B$5*(B62-$B$3)+B62 |
| =A63+$B$4 | =$B$4*$B$5*(B63-$B$3)+B63 |
| =A64+$B$4 | =$B$4*$B$5*(B64-$B$3)+B64 |
| =A65+$B$4 | =$B$4*$B$5*(B65-$B$3)+B65 |
| =A66+$B$4 | =$B$4*$B$5*(B66-$B$3)+B66 |
| =A67+$B$4 | =$B$4*$B$5*(B67-$B$3)+B67 |
| =A68+$B$4 | =$B$4*$B$5*(B68-$B$3)+B68 |
| =A69+$B$4 | =$B$4*$B$5*(B69-$B$3)+B69 |
| =A70+$B$4 | =$B$4*$B$5*(B70-$B$3)+B70 |
| =A71+$B$4 | =$B$4*$B$5*(B71-$B$3)+B71 |
| =A72+$B$4 | =$B$4*$B$5*(B72-$B$3)+B72 |
| =A73+$B$4 | =$B$4*$B$5*(B73-$B$3)+B73 |
| =A74+$B$4 | =$B$4*$B$5*(B74-$B$3)+B74 |
| =A75+$B$4 | =$B$4*$B$5*(B75-$B$3)+B75 |
| =A76+$B$4 | =$B$4*$B$5*(B76-$B$3)+B76 |
| =A77+$B$4 | =$B$4*$B$5*(B77-$B$3)+B77 |
| =A78+$B$4 | =$B$4*$B$5*(B78-$B$3)+B78 |
| =A79+$B$4 | =$B$4*$B$5*(B79-$B$3)+B79 |
| =A80+$B$4 | =$B$4*$B$5*(B80-$B$3)+B80 |
| =A81+$B$4 | =$B$4*$B$5*(B81-$B$3)+B81 |
| =A82+$B$4 | =$B$4*$B$5*(B82-$B$3)+B82 |
| =A83+$B$4 | =$B$4*$B$5*(B83-$B$3)+B83 |
| =A84+$B$4 | =$B$4*$B$5*(B84-$B$3)+B84 |
| =A85+$B$4 | =$B$4*$B$5*(B85-$B$3)+B85 |
| =A86+$B$4 | =$B$4*$B$5*(B86-$B$3)+B86 |
| =A87+$B$4 | =$B$4*$B$5*(B87-$B$3)+B87 |
| =A88+$B$4 | =$B$4*$B$5*(B88-$B$3)+B88 |
| =A89+$B$4 | =$B$4*$B$5*(B89-$B$3)+B89 |
| =A90+$B$4 | =$B$4*$B$5*(B90-$B$3)+B90 |
| =A91+$B$4 | =$B$4*$B$5*(B91-$B$3)+B91 |
| =A92+$B$4 | =$B$4*$B$5*(B92-$B$3)+B92 |
| =A93+$B$4 | =$B$4*$B$5*(B93-$B$3)+B93 |
| =A94+$B$4 | =$B$4*$B$5*(B94-$B$3)+B94 |
| =A95+$B$4 | =$B$4*$B$5*(B95-$B$3)+B95 |
| =A96+$B$4 | =$B$4*$B$5*(B96-$B$3)+B96 |
| =A97+$B$4 | =$B$4*$B$5*(B97-$B$3)+B97 |
| =A98+$B$4 | =$B$4*$B$5*(B98-$B$3)+B98 |
| =A99+$B$4 | =$B$4*$B$5*(B99-$B$3)+B99 |
| =A100+$B$4 | =$B$4*$B$5*(B100-$B$3)+B10 |
| =A101+$B$4 | =$B$4*$B$5*(B101-$B$3)+B10 |
| =A102+$B$4 | =$B$4*$B$5*(B102-$B$3)+B10 |
| =A103+$B$4 | =$B$4*$B$5*(B103-$B$3)+B10 |

| | |
|---|---|
| =A104+$B$4 | =$B$4*$B$5*(B104-$B$3)+B10 |
| =A105+$B$4 | =$B$4*$B$5*(B105-$B$3)+B10 |
| =A106+$B$4 | =$B$4*$B$5*(B106-$B$3)+B10 |
| =A107+$B$4 | =$B$4*$B$5*(B107-$B$3)+B10 |
| =A108+$B$4 | =$B$4*$B$5*(B108-$B$3)+B10 |